DEMYSTIFYING OPTION PRICING THEORY

How to Use Options in Every Phase of the Valuation Process

VSCPA Forensic & Valuation Services Conference
September 27, 2023
2:45pm – 4:25pm

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INTRODUCTION

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Learning Objectives

- **Option Pricing Theory Overview**
- **This session will explore the usage of option pricing models (OPMs) to value securities in a complex capital structure (e.g., in a stock-based compensation engagement), as well as their application for estimating discounts for lack of marketability.**
- **Using option pricing theory to test management's projections.**
- **Participants will learn about the various types of OPMs available, including the applicability and limitations of each**
- **A case study will be utilized to demonstrate how these models are used in practice.**

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OPTION PRICING THEORY

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Definition

- An option is a financial derivative that gives the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined price within a specified time frame.
    - Call option – Right to Buy
    - Put option – Right to Sell
- The value of the option is having the ***right or privilege*** to execute the contract. That is what you're buying, selling, or giving to employees.

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BLACK-SCHOLES

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History Lesson

- **Pre-1970s**: Before the Black-Scholes model, there was no universally accepted method for pricing options. Traders mostly relied on intuition and simplified models.
- **Early Research**: In the early 1970s, academics and practitioners began looking at ways to calculate the value of options in a more systematic and reliable manner.
- **Fisher Black and Myron Scholes**: Fisher Black and Myron Scholes, both finance scholars, began collaborating on developing a mathematical formula to price options.
- **Robert Merton's Contribution**: Around the same time, Robert C. Merton, another finance scholar, was independently working on similar problems. Merton later added crucial extensions to the original Black-Scholes work.
- **Assumptions**: The model they developed was based on several assumptions, including constant volatility and interest rates, and it didn't account for taxes or transaction costs.
- **Publication**: In 1973, Black and Scholes published their groundbreaking paper, "The Pricing of Options and Corporate Liabilities," in the Journal of Political Economy. The paper presented the Black-Scholes formula for pricing European call and put options.
- **Immediate Impact**: The model was immediately popular and started being used by options traders. It led to a more standardized and liquid options market.
- **Nobel Prize**: In 1997, Myron Scholes and Robert Merton were awarded the Nobel Prize in Economic Sciences for their contributions to the field of financial economics. Fisher Black, who had passed away by that time, was not eligible for the prize, but his contributions were acknowledged.
- **Further Developments**: Over the years, the model has been adapted and extended to account for other types of options, varying interest rates, and more complex financial instruments.
- **Continuing Legacy**: Despite its limitations and assumptions, the Black-Scholes model remains one of the most important and widely-used concepts in modern finance.

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Concepts and Real Options

- An option has value or "in the money" if it crosses a certain threshold.
- If we cross that threshold or breakpoint, we get to play in the sandbox. If we don't, we don't get to have any fun.
- Don't Get Lost in the Sauce….. More on this later.

Black-Scholes Formula (European Call Option):

$$C = S \cdot e^{-q \cdot T} \cdot N(d_1) - K \cdot e^{-r \cdot T} \cdot N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right) \cdot T}{\sigma \cdot \sqrt{T}}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{T}$$

$N(d)$ is the cumulative distribution function of the standard normal distribution.

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Inputs to Black-Scholes

- S: Price of the underlying asset
- K: Exercise or strike price of the option (aka the Threshold)
- T: Time to expiration of the option
- r: Risk-free interest rate
- σ: Volatility of the underlying asset
- q: Dividend yield (if applicable)

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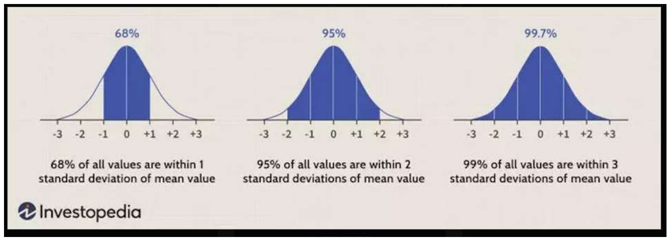
Normal Distribution

- The concept of normal distribution is central to the Black-Scholes model as it helps calculate the probabilities associated with the future prices of the underlying asset.



- In the Black-Scholes model, stock prices are assumed to follow a log-normal distribution, which means the stock returns are normally distributed.

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Limitations of the Black Scholes Model

Limitation	Description
Constant Volatility	Assumes constant volatility of the underlying asset, while in reality, volatility varies over time.
Log-normal Distribution of Stock Prices	Assumes log-normally distributed stock prices, ignoring the possibility of skewness and kurtosis in real asset returns.
Constant Interest Rates	Assumes a constant risk-free interest rate, which can vary in real-world scenarios due to economic conditions and monetary policy.
No Dividends	The original model does not account for dividends, limiting its application to certain assets. Extensions have been made to include dividend yield.
European Options Only	Specifically designed for European options, which can only be exercised at expiration, not directly applicable to American options.
Liquidity and Transaction Costs	Does not consider transaction costs or the impact of liquidity on option pricing, which can be significant in real markets.
Counterparty Risk	Ignores the credit risk of the counterparty, a significant factor especially in over-the-counter (OTC) markets.
Market Sentiments and Behavioral Factors	Does not account for market sentiments, irrational behavior, or other market anomalies, which can influence option prices.

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CASE STUDY 1

EXPLAIN THIS TO A 5TH GRADER

Let’s Describe This to a 5th Grader

Background

Imagine Jimmy really wants a new video game that's coming out in 6 months. Betty offers to make a deal with Jimmy: Jimmy can pay Betty $5 now, and in return, Betty will give you the option to buy the game for $50 when it's released. This $5 is sort of like a reservation fee; you're paying for the right to buy the game at a fixed price later on.

In this example, the video game is like the "underlying asset". The $5 Billy paid upfront is the price of the "option," and the $50 Billy can pay later is called the "strike price."

Did Jimmy Get a Good Deal?

1. **Price of the Video Game (S)**: If the game ends up being super popular and sells out everywhere, the price might go up to $70 or even $100. In that case, your option to buy it for $50 is a great deal! But if the game isn't popular and stores are selling it for $40, then your option isn't as valuable.
2. **Time (T)**: You have six months to decide whether you want to buy the game at that $50 price. If you learn after two months that the game is amazing, you might feel great about your deal. On the other hand, if the game gets bad reviews, you might decide not to buy it.
3. **Popularity and Hype (σ)**: If everyone expects this game to be the next big thing, the price to reserve that $50 deal might go up. This is sort of like how stock prices go up and down based on how much people want them.
4. **Money in the Bank (r and q)**: Let's say you could earn a little interest if you kept your $5 in a piggy bank or a savings account. That's another thing to consider because you're giving up that interest when you pay for the option.

Basic idea : you're paying a bit of money now (or giving up something, like time and effort) for the chance to make a good deal later.

CASE STUDY 2

APPLICATION OF BLACK-SCHOLES MODEL: EMPLOYEE OPTION POOL

Application of Black-Scholes Model: Employee Option Pool

› Black-Scholes can be used to determine the value of stock options issued for financial reporting and tax purposes:

Assumptions	Fair Value
Value of Common Stock (S)	$6.25
Strike Price (X)	$7.50
Expected dividend yield (d)	0.00%
Expected dividend yield (continuous) (dy)	0.00%
Time to expiration (t)	5.00
Volatility (σ)	35.00%
Risk-free rate (rf)	3.66%
Risk-free interest rate (continuous) (r)	3.59%
d1	0.39
Normal distribution of d1 (N(d1))	0.65
d2	(0.39)
Normal distribution of d2 (N(d2))	0.35
Fair value of stock option	$1.8971
Total options granted in 2022	225,000
Fair value of stock option	$1.8971
Total fair value of 2022 stock options	$426,843

Excel Formula: Value of Call Option = S*EXP(-dy*t)*(NORMSDIST(d1))-X*EXP(-r*t)*(NORMSDIST(d2))

Where d1 =(LN(S/X)+(r-dy+(v^2/2))*t)/(v*SQRT(t)) and d2 =d1-(v*SQRT(t))

Application of Black-Scholes Model: Employee Option Pool

Calculated using Black-Scholes-Merton Formula.

$$Call\ option\ price = Se^{-dyt}N(d_1) - Xe^{-rt}N(d_2)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - dy + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$r = \ln(1 + rf)$$

$$dy = \ln(1 + d)$$

r = continuously compounded value of 1 plus risk-free interest rate
rf = risk-free interest rate
d = expected dividend yield
dy = continuously compounded value of 1 plus expected dividend yield
ln = natural logarithm
S = current common stock price (asset value)
X= exercise price of option
σ = volatility of underlying asset
t = time to expiration (liquidation) in years
e = exponential growth constant function
N = standard normal cumulative distribution function

Source: *Understanding Business Valuation*, Gary R. Trugman CPA/ABV, FASA, MVS.

Complex Capital Structures

- Many early-stage entities are financed through a combination of different types of securities (e.g., common equity, preferred equity, convertible debt).
- OPM can be used to allocate total equity value to multiple classes of securities.
- Treats each class of equity as a call option on the total equity point. Therefore, you are considering multiple "thresholds" or commonly called "breakpoints"
- Comprehensive example to recreate yourself
  - https://www.cpajournal.com/2020/09/09/valuing-securities-using-the-option-pricing-method/

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CASE STUDY 3

ALLOCATING EQUITY VALUE TO DIFFERENT CLASSES OF SECURITIES

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Allocating Equity Value to Different Classes of Securities – Breakpoints

› First step, determine breakpoints of change in distribution of allocable value:

Event	Participating Class	Participating Units	Breakpoint
1 FMV per Common Unit less than $10.00	Common	3,125,273	$0
2 FMV per Common Unit greater than $10.00	Common and NPIs	4,217,060	$31,252,731

› Based upon breakpoints above, the securities participate in the allocable value as follows:

	Breakpoint	
Allocation %	Event 1	Event 2
Common Units	100.00%	74.11%
Net Profit Interests	0%	25.89%
Total	100.00%	100.00%

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Allocating Equity Value to Different Classes of Securities – OPM

› Black-Scholes is next used to determine the incremental value of each breakpoint:

	Breakpoint	
Assumptions	1	2
Value of underlying Asset (S)	$24,718,269	$24,718,269
Strike Price (Breakpoint) (X)	$0	$31,252,731
Expected dividend yield (d)	0%	0%
Expected dividend yield (continuous) (dy)	0%	0%
Time to expiration (t)	5.00	5.00
Volatility (v)	60%	60%
Risk-free rate (rf)	1.62%	1.62%
Risk-free interest rate (continuous) (r)	1.61%	1.61%
d1	28.87	0.56
Normal distribution of d1 (N(d1))	1.00	0.71
d2	27.52	(0.79)
Normal distribution of d2 (N(d2))	1.00	0.22
Value of Common Unit as option	$24,718,269	$11,341,566
Incremental Value	$13,376,703	$11,341,566

Excel Formula: Value of Call Option = S*EXP(-dy*t)*(NORMSDIST(d1))-X*EXP(-r*t)*(NORMSDIST(d2))

Where d1 =(LN(S/X)+(r-dy+(v^2/2))*t)/(v*SQRT(t)) and d2 =d1-(v*SQRT(t))

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Allocating Equity Value to Different Classes of Securities – OPM

Calculated using Black-Scholes-Merton Formula.

r = LN(1+rf)

dy = LN(1+d)

d1 = LN(S/X) + (r-dy + (v^2/2))*t / v*SQRT(t)

d2 = d1 - (v*SQRT(t))

Value of common stock as option =
S * EXP(-dy * t) * NORMSDIST(d1) - X * EXP(-r * t) * NORMSDIST(d2)

Where:
r = continuously compounded value of 1 plus risk-free
rf = risk-free interest rate
d = expected dividend yield
dy = continuously compounded value of 1 plus expected
LN = natural logarithm
S = current stock price (asset value)
X= exercise price of option (breakpoint)
v = volatility of underlying asset
t = time to expiration (liquidation) in years
EXP = exponential growth constant function
NORMSDIST = standard normal cumulative distribution function

Source: *Understanding Business Valuation*, Gary R. Trugman CPA/ABV, FASA, MVS.

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Allocating Equity Value to Different Classes of Securities – Allocation of Value

- As calculated previously, the securities participate in incremental allocable values as follows:

	Breakpoint	
Allocation %	Event 1	Event 2
Common Units	100.00%	74.11%
Net Profit Interests	0%	25.89%
Total	100.00%	100.00%

- Therefore, the incremental values are allocated between the securities as seen below:

Allocation of Value $	Event 1	Event 2	Total share class value
Common Units	$13,376,703	$8,405,261	$21,781,964
Net Profit Interests	0	2,936,305	2,936,305
Total	$13,376,703	$11,341,566	$24,718,269

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Allocating Equity Value to Different Classes of Securities – Price Volatility

- Annualized price volatility was estimated for the subject company based upon weekly returns for guideline public companies as follows:

Ticker	Company	Price Volatility 5 Year
LNW	Light & Wonder, Inc.	75.42%
CHDN	Churchill Downs Incorporated	42.41%
EVRI	Everi Holdings Inc.	74.33%
BYD	Boyd Gaming Corporation	51.63%
PENN	PENN Entertainment, Inc.	71.03%
IGT	International Game Technology PLC	59.49%
	Minimum	42.41%
	25th percentile	53.60%
	Mean	62.39%
	Median	65.26%
	75th percentile	73.51%
	Maximum	75.42%
	Selected Price Volatility	**60.00%**

Allocating Equity Value to Different Classes of Securities – Other Volatilities

- Depending on the complexity of the capital structure, other types of volatility may be appropriate.
  - For example, asset volatilities (shown below) may be used if the subject company includes convertible debt. If multiple types of equity securities are involved (e.g., preferred, common, warrants, options, etc.), then class volatility may be appropriate.

Ticker	Company	Market Capitalization	Interest Bearing Debt	MVIC	Dividend Yield (Continuous)	Equity Volatility 5 Year	Risk-Free Rate (Continuous) 5 Year
LNW	Light & Wonder, Inc.	$6,937	$8,749	$15,686	0.00%	75.42%	1.61%
CHDN	Churchill Downs Incorporated	8,915	1,968	10,883	0.30%	42.41%	1.61%
EVRI	Everi Holdings Inc.	1,244	999	2,243	0.00%	74.33%	1.61%
BYD	Boyd Gaming Corporation	6,276	3,932	10,208	1.01%	51.63%	1.61%
PENN	PENN Entertainment, Inc.	3,472	11,605	15,077	0.00%	71.03%	1.61%
IGT	International Game Technology PLC	$6,347	$6,864	$13,211	2.49%	59.49%	1.61%

Ticker	N(d1) 5 Year	N(d2) 5 Year	Call Option Value 5 Year	Implied Debt Value 5 Year	Equity Implied Asset Values 5 Year	Equity Implied Asset Volatilities 5 Year
LNW	0.885	0.211	$12,171	$3,515	$10,452	56.58%
CHDN	0.989	0.516	9,666	1,217	10,131	37.73%
EVRI	0.908	0.225	1,829	415	1,659	61.42%
BYD	0.921	0.407	7,458	2,751	9,026	39.00%
PENN	0.834	0.225	10,158	4,919	8,391	35.25%
IGT	0.875	0.325	$8,154	$5,058	11,405	37.83%

DISCOUNTS FOR LACK OF MARKETABILITY

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Alternative Option Models to Calculate DLOMs

- "Investors in fully marketable securities own valuable liquidity options that enable them to cost-effectively realize gains, avoid losses, adequately diversify their holdings, and capitalize on new, profitable investment opportunities. Investors in nonmarketable securities, however, forego these valuable liquidity options and suffer an opportunity cost as a result. As a consequence of the market frictions imposed on nonmarketable stock, investors demand sizable discounts to compensate for the opportunity costs of being restricted from reselling. **In theory, an investor could purchase a hypothetical put option to sell their nonmarketable stock. The price of this option calculated using option pricing models can be used as an estimate of the opportunity costs in terms of lost economic payoffs during the restriction period for the security** [emphasis added]."

Hitchner, James R., et al. Discount for Lack of Marketability Guide and Toolkit. Ventnor City, NJ: Valuation Products and Services, LLC. 2017.

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Alternative Option Models to Calculate DLOMs

- The most widely known and recognized option pricing models for purposes of estimating a DLOM include the following:
  - Finnerty – Average-Strike Put Option Model
  - Ghaidarov – Adjusted Arithmetic Average-Strike Option Model
  - Chaffe – Option Pricing Model (based on Black Scholes)
  - Longstaff – Upper Bound Lookback Put Option Model

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Alternative Option Models to Calculate DLOMs

Finnerty - Average-Strike Put Option Model

$$DLOM = V_0 e^{-qt}\left[N\left(\frac{v\sqrt{T}}{2}\right) - N\left(-\frac{v\sqrt{T}}{2}\right)\right]$$

$$v\sqrt{T} = \left[\sigma^2 T + ln\left[2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right] - 2ln\left[e^{\sigma^2 T} - 1\right]\right]^{\frac{1}{2}}$$

Where:

V_0 = Current freely traded value of the security
e = Exponential growth constant
q = Dividend yield of the security
T = Time of restriction (i.e., holding period)
N = Cumulative normal distribution function
σ = Volatility
ln = Natural logarithm

Excel Formula: DLOM =EXP(-q*T)*(NORMSDIST(v√T/2)-NORMSDIST(-v√T/2))

where v√T =(σ^2*T+LN(2*(EXP(σ^2*T)- σ^2*T-1))-2*LN(EXP(σ^2*T)-1))^(1/2)

Hitchner, James R., et al. Discount for Lack of Marketability Guide and Toolkit. Ventnor City, NJ: Valuation Products and Services, LLC. 2017.

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Alternative Option Models to Calculate DLOMs

Ghaidarov – Adjusted Arithmetic Average-Strike Option Model

$$D = e^{-qt}V(0)\left[2N\left(\frac{V_T}{2}\right) - 1\right]$$

$$V_T^2 = ln\left[2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right] - 2ln[\sigma^2 T]$$

Where:

D = Discount for lack of marketability
e = Exponential growth constant
q = Continuous dividend yield
T = Holding period restriction (in years)
V(0) = Initial stock price
N = Cumulative standard normal distribution function
σ = Volatility of the stock price

Excel Formula: DLOM = EXP(-q*T)*(2*NORMSDIST(SQRT(v²T)/2)-1)

Where v²T =LN(2*(EXP(σ^2*T)- σ^2*T-1))-2*LN(σ^2*T)

Hitchner, James R., et al. Discount for Lack of Marketability Guide and Toolkit. Ventnor City, NJ: Valuation Products and Services, LLC. 2017.

Alternative Option Models to Calculate DLOMs

Chaffe - Option Pricing Model

$$DLOM = Ke^{-rt}N(-d_2) - S_0 e^{-q}\, N(-d_1)$$

$$d_1 = \frac{(ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right) * T}{(\sigma\sqrt{T})}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Where:

K = Strike price of the option
r = Risk-free rate
T = Time of restriction (i.e., holding period)
σ = Volatility
N = Cumulative normal distribution function
ln = Natural logarithm
q = Continuously compounded dividend yield
e = Exponential growth constant

Excel Formula: DLOM =(EXP(-LN(1+r)*T))*NORMSDIST(-d2)-EXP(-LN(1+q)*T)*NORMSDIST(-d1)

Where d1 =(LN(1)+((LN(1+r)-(LN(1+q))+σ^2/2)*T)/(σ*T^0.5)) and d2 =d1-σ*T^0.5

Hitchner, James R., et al. Discount for Lack of Marketability Guide and Toolkit. Ventnor City, NJ: Valuation Products and Services, LLC. 2017.

Alternative Option Models to Calculate DLOMs

Longstaff - Upper Bound Lookback Put Option Model

$$D*(T) = V_0\left[\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right)+\sqrt{\frac{\sigma^2 T}{2\pi}}\;exp\left(-\frac{\sigma^2 T}{8}\right)-1\right]$$

Where:

V_0 = Value of the security
T = Time of restriction (i.e., holding period)
σ = Volatility
N = Cumulative normal distribution function
exp = Exponential growth constant

Excel Formula: Adjusted DLOM =DLOM/(1+DLOM)

Where DLOM=(2+(σ^2*T/2))*NORMSDIST(SQRT(σ^2*T)/2)+SQRT(σ^2*T/(2*PI()))*EXP(-(σ^2*T/8))-1

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CASE STUDY 4

ALTERNATIVE OPTION MODELS TO CALCULATE DLOMS

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Alternative Option Models to Calculate DLOMs

- The range of discounts for lack of marketability indicated by these alternative option models for our subject company were as follows:

	Finnerty	Ghaidarov	Chaffe (Protective Put)	Longstaff
Option pricing inputs - 5 year holding period				
Volatility	60.0%	60.0%	60.0%	60.0%
Dividend yield	0.0%	0.0%	0.0%	N/A
Holding period (years)	5.0	5.0	5.0	5.0
Risk-free rate	N/A	N/A	1.62%	N/A
Indicated discount for lack of marketability	25.8%	32.3%	44.1%	61.5%

MONTE CARLO SIMULATIONS

Monte Carlo Simulations v. OPM

Aspect	Option Pricing Model	Monte Carlo Simulation
Definition	Mathematical model used to calculate the theoretical value of an option.	Statistical technique to model the probability of different outcomes in a process with random variables.
Examples	Black-Scholes Model, Binomial Model	N/A (Monte Carlo is a technique, not a specific model)
Approach	Deterministic: Calculates a single, definite price based on given inputs.	Stochastic: Uses random sampling to estimate outcomes and their probabilities.
Flexibility	Limited flexibility, operates under specific assumptions.	Highly flexible, can model a wide range of scenarios and complex derivatives.
Computational Intensity	Generally less computationally intensive.	Can be computationally intensive due to a large number of simulations.
Applicability	Suitable for simpler, well-defined option types.	Suitable for complex options and derivatives where traditional models may not apply.

Monte Carlo Simulations

- Build your DCF model
  - Once you build your model, you're 90% complete in setting up the simulation
- Identify the assumptions you would like to simulate and define the upper and lower bounds
  - Revenue growth
  - Operating margins
- Best to use when estimations for growth are uncertain or volatile.

CASE STUDY 5

SCENARIO ANALYSIS WITH MANAGEMENT PROJECTIONS USING MONTE CARLO

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Scenario Analysis with Management Projections Using Monte Carlo

› Monte Carlo simulations can be utilized to check the sensitivity of management's projections on overall value. The projections below use simulations to test the effect of changes in revenue growth and operating expenses as variables (see next slide).

	Projected Financial Statements										
											Terminal
	12/31/22	12/31/23	12/31/24	12/31/25	12/31/26	12/31/27	12/31/28	12/31/29	12/31/30	12/31/31	12/31/32
Revenue	$20,227,490	$26,844,518	$33,730,166	$41,824,032	$49,629,154	$59,880,763	$67,877,739	$73,254,421	$79,113,108	$82,856,265	$85,323,953
Percent increase	*0.185*	*0.327*	*0.257*	*0.240*	*0.187*	*0.207*	*0.134*	*0.079*	*0.080*	*0.047*	*0.030*
Cost of sales	3,034,124	4,026,678	5,059,525	6,273,605	7,444,373	8,982,114	10,181,661	10,988,163	11,866,966	12,428,440	12,798,593
Percent of revenue	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*	*0.15*
Gross profit	17,193,367	22,817,840	28,670,641	35,550,427	42,184,781	50,898,649	57,696,078	62,266,257	67,246,141	70,427,825	72,525,360
Percent of revenue	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*	*0.85*
Operating expenses											
Operating expenses	10,676,679	12,535,873	11,107,607	11,600,082	13,045,840	17,654,529	20,343,279	16,904,112	19,471,811	19,233,396	16,381,983
Depreciation expense	2,141,848	3,967,157	5,945,052	8,435,421	11,409,066	12,701,550	14,654,560	16,679,447	18,431,960	19,851,757	22,457,894
Amortization expense	1,468,247	2,006,036	2,672,901	3,503,737	4,528,784	5,712,636	7,015,063	6,926,618	7,851,397	8,698,999	10,838,071
Total operating expenses	14,286,774	18,509,066	19,725,561	23,539,240	28,983,690	36,068,715	42,012,901	40,510,177	45,755,168	47,784,152	49,677,948
Percent of revenue	*0.71*	*0.69*	*0.58*	*0.56*	*0.58*	*0.60*	*0.62*	*0.55*	*0.58*	*0.58*	*0.58*
Operating income	2,906,593	4,308,774	8,945,080	12,011,187	13,201,090	14,829,934	15,683,177	21,756,080	21,490,973	22,643,673	22,847,411
Percent of revenue	*0.14*	*0.16*	*0.27*	*0.29*	*0.27*	*0.25*	*0.23*	*0.30*	*0.27*	*0.27*	*0.27*
Other income (expense)											
Interest expenses	(958,262)	(1,178,343)	(1,452,743)	(1,611,914)	(1,716,669)	(1,809,644)	(1,892,251)	(1,981,532)	(2,078,020)	(2,182,292)	(2,247,287)
Other expenses	(360,000)	(360,000)	(360,000)	(360,000)	(360,000)	(360,000)	(360,000)	(360,000)	(360,000)	(360,000)	(370,722)
Total other income (expense)	(1,318,262)	(1,538,343)	(1,812,743)	(1,971,914)	(2,076,669)	(2,169,644)	(2,252,251)	(2,341,532)	(2,438,020)	(2,542,292)	(2,618,009)
Percent of revenue	*(0.07)*	*(0.06)*	*(0.05)*	*(0.05)*	*(0.04)*	*(0.04)*	*(0.03)*	*(0.03)*	*(0.03)*	*(0.03)*	*(0.03)*
Adjusted net income (loss) before taxes	1,588,331	2,770,431	7,132,337	10,039,273	11,124,421	12,660,290	13,430,926	19,414,548	19,052,953	20,101,381	20,229,403
Percent of revenue	*0.08*	*0.10*	*0.21*	*0.24*	*0.22*	*0.21*	*0.20*	*0.27*	*0.24*	*0.24*	*0.24*

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Scenario Analysis with Management Projections Using Monte Carlo

- The assumptions below for revenue growth and operating expenses (as a % of revenue) and a normal probability distribution were used to run the Monte Carlo simulations. Different types of probability distributions may be appropriate (e.g., lognormal, triangular, uniform, etc.) depending on the variable (see next slide).

	Distributions			
	Revenue Growth		Opex (% of Revenue)	
Year	Mean	Std. Dev.	Mean	Std. Dev.
1	18.4%	2.30%	54.50%	6.81%
2	30.9%	3.86%	44.70%	5.59%
3	25.6%	3.20%	38.27%	4.78%
4	25.6%	3.21%	32.54%	4.07%
5	24.4%	3.05%	27.48%	3.44%
6	20.0%	2.50%	24.06%	3.01%
7	15.0%	1.88%	21.97%	2.75%
8	10.0%	1.25%	20.98%	2.62%
9	7.5%	0.94%	20.50%	2.56%
10	5.0%	0.63%	20.50%	2.56%
11	3.0%	0.38%	20.50%	2.56%

Random Variables from Distributions				
Year	Revenue	Revenue growth	Opex	Opex (% of Revenue)
Y0	17,066,520			
1	20,227,490	18.52%	10,676,679	52.78%
2	26,844,518	32.71%	12,535,873	46.70%
3	33,730,166	25.65%	11,107,607	32.93%
4	41,824,032	24.00%	11,600,082	27.74%
5	49,629,154	18.66%	13,045,840	26.29%
6	59,880,763	20.66%	17,654,529	29.48%
7	67,877,739	13.35%	20,343,279	29.97%
8	73,254,421	7.92%	16,904,112	23.08%
9	79,113,108	8.00%	19,471,811	24.61%
10	82,856,265	4.73%	19,233,396	23.21%
11	85,323,953	2.98%	16,381,983	19.20%

- Management's base case scenario or an average of various projections can be used as the mean. Standard deviation can be estimated based on historical results or difference in management's scenarios

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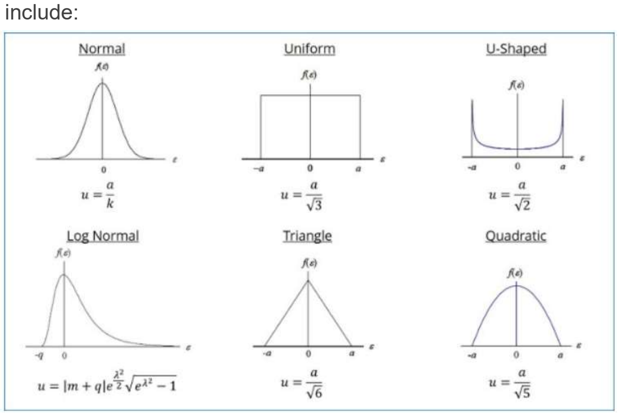
Scenario Analysis with Management Projections Using Monte Carlo

- A few different types of distributions that may be appropriate depending on the variable include:



https://www.isobudgets.com/probability-distributions-for-measurement-uncertainty/

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Scenario Analysis with Management Projections Using Monte Carlo

- Monte Carlo simulations are frequently used in financial reporting engagements (e.g., valuation of contingent consideration/earnouts). The forecast example on the right ran simulations using pounds shipped and price per pound as variables in determining the possible outcomes of the present value of an earnout.
- When Monte Carlo simulations are used to analyze financial projections, consideration should be given to reducing to discount rate used in DCF due to shifting of risk analysis.

		Qrt 12 02/29/20	Qrt 13 05/31/20	Qrt 14 08/31/20	Qrt 15 11/30/20	Qrt 16 02/28/21
Estimated quarterly pounds shipped in excess of threshold	Base - Qrt 11					
High	6,906,687	7,562,113	8,566,546	9,376,042	8,408,797	9,462,807
Moderate	6,906,687	7,341,857	8,081,648	8,601,874	7,507,855	8,157,592
Low	6,906,687	7,121,601	7,596,749	7,827,705	6,606,912	6,852,377
Quarterly growth rates of estimated pounds shipped in excess of threshold						
High		9.5%	13.3%	9.4%	(10.3%)	12.5%
Moderate		6.3%	10.1%	6.4%	(12.7%)	8.7%
Low		3.1%	6.7%	3.0%	(15.6%)	3.7%
Mean		6.3%	10.0%	6.3%	(12.9%)	8.3%
Standard deviation		2.6%	2.7%	2.6%	2.2%	3.6%
Estimated quarterly growth rates per Monte Carlo Simulation		**6.3%**	**10.1%**	**6.4%**	**(12.7%)**	**8.7%**
	Base - Qrt 11					
Quarterly pounds shipped (forecast)	6,906,687	7,341,857	8,081,648	8,601,874	7,507,855	8,157,592
Quarterly growth rates of estimated price per pound - blended	Base - Qrt 11					
High	$1.98	$2.21	$2.23	$2.29	$2.34	$2.44
Moderate	$1.98	$2.14	$2.10	$2.10	$2.09	$2.10
Low	$1.98	$2.08	$1.98	$1.91	$1.84	$1.76
Quarterly growth rates of estimated price per pound						
High		11.3%	1.0%	2.5%	2.5%	4.0%
Moderate		8.1%	(1.8%)	(0.3%)	(0.2%)	0.4%
Low		4.8%	(4.9%)	(3.5%)	(3.5%)	(4.2%)
Mean		8.1%	(1.9%)	(0.5%)	(0.4%)	0.1%
Standard deviation		2.6%	2.4%	2.5%	2.5%	3.3%
Estimated quarterly growth rates per Monte Carlo Simulation		**8.1%**	**(1.8%)**	**(0.3%)**	**(0.2%)**	**0.4%**
	Base - Qrt 11					
Quarterly price per pound (forecast) - blended price	$1.98	$2.14	$2.10	$2.10	$2.09	$2.10
Net sales		15,740,950	17,008,510	18,041,917	15,710,705	17,138,496
Earn out at 3%		$472,229	$510,255	$541,258	$471,321	$514,155
Total undiscounted earn out	$2,509,217					
Present value factor		0.9867	0.9747	0.9627	0.9511	0.9397
Present value of earn out payments		$465,958	$497,326	$521,095	$448,278	$483,169
Total present value of earn out for base case		$2,415,826				
Total present value of earn out for mean		$2,403,599				
Total present value of earn out for median		$2,400,497				

Monte Carlo Simulation Tools

- **Excel**
  - Excel, with its built-in functions and Data Table feature, can perform basic Monte Carlo simulations. Additionally, add-ins like @RISK can significantly enhance its capabilities for Monte Carlo simulation.
  - Best for simpler models and educational purposes due to its ease of use
- **RiskAMP**
  - Compatible with Excel
  - Free trial versions and several whitepapers and how-to guides
- **Crystal Ball**
  - Oracle Crystal Ball is an Excel add-in that is used for predictive modeling, forecasting, simulation, and optimization.
- **Palisade @RISK**
  - @RISK is an Excel add-in used for risk analysis using Monte Carlo simulation.

Other sources

- **Damodaran**
  - Monte Carlo Simulations in Finance & Investing
  - https://www.youtube.com/watch?v=eXhCXobViJc

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